Imaging OCR B (AS)

Pixels, bits and bytes

Think of a pixel as a cell that can store energy.
A bit is a 'piece' of information – here are two alternative values, it can either be 0 or 1.

Information

Information in an image = Number of pixels x bits/pixel

In general there are 2^I alternatives if there are I bits of information.
A byte is a number (I) of bits, in fact a byte is now defined as 8 bits – there are 2^8 alternatives.

Information

An 8-bit byte 'contains' 2^8 = 256 bits

Example 1

A 2-bit byte 'contains' 2^2 = 4 bits
A 4-bit byte 'contains' 2^4 = 16 bits

Example 2

If we have a digital camera image that is 10 mega pixels with each pixel requiring 3 bytes (3x8 = 24 bits) then the image will need 30 Mbytes of storage.

The use of logs and logs to the base 2

Logs to the base to are related to logs to the base ten by the formula:

Information

$$\log_2 A = \log_{10} A / \log_{10} 2$$

Information

Amount of information (I) provides $N = 2^I$ alternatives

$I = \log_2 N$

Example 3

(a) $\log_2 8 = \log_{10} 8/\log_{10} 2 = 0.9031/0.3010 = 3$.
This means that 8 is 2 raised to the power of 3 i.e. 3 cubed.
Of course this also works for other, less simple, numbers.
Example 2: $\log_2 15 = \log_{10} 15/\log_{10} 2 = 1.176/0.3010 = 3.322$
This means that 15 is 2 raised to the power of 3.322

(b) a text message with lower and upper case letters, numbers 0 to 9 and 12 different punctuation marks. This is a total of 74 characters.
Number of bits must be $\geq \log 74/\log 2 = 6.2$
Therefore 7 bits are needed

Notice that the answer must be a whole number!

Rate of transmission of information

For a digitised signal the more samples per second and the greater number of bits within each sample the greater the rate of transmission of information.

Information

Rate of transmission of information

= Number of samples per second x bits per sample

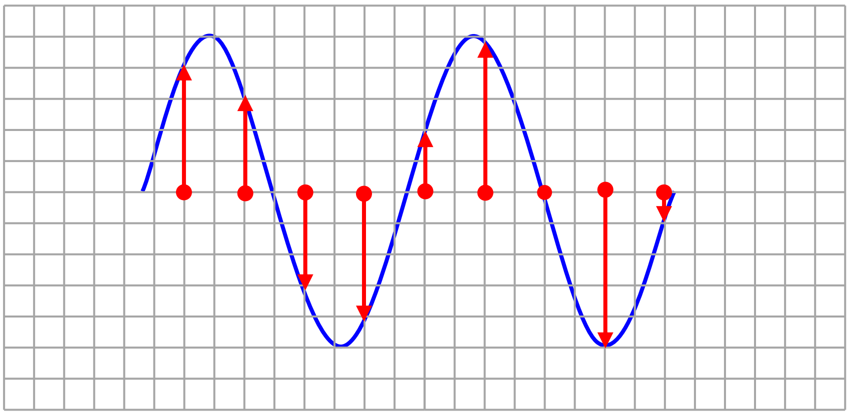
You can think of the quality of an image that has many shades of grey, the more shades (the more bits per sample) the more information is transmitted.

Example 4

A signal is sampled at 44 100 Hz with 16 bits per sample.
Calculate:
The rate at which the data is processed (rate of transmission of information)
Rate of information transfer = 44100x16 = 7.056×10^5 bit (s^{-1})

Sampling

To digitize a signal you take its value at **equal** spaced points along it in time – known as sampling. This is shown by the series of red arrows and discs on the following graph. Sampling means that the digital intensity of the wave at these times is taken.



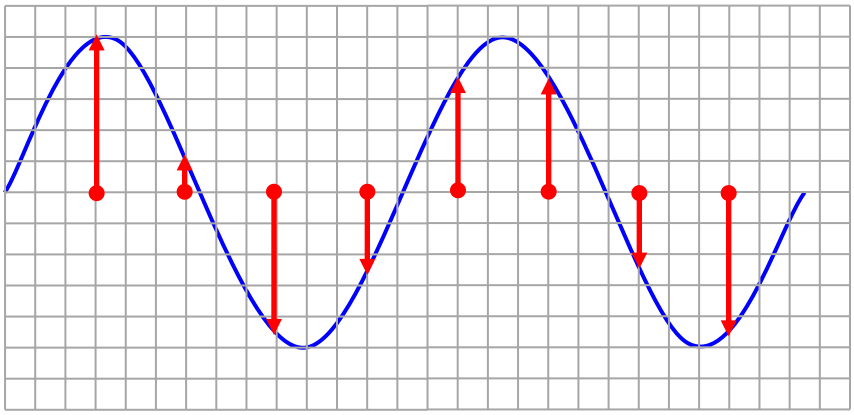
Rate of sampling

To reproduce the signal from its samples the sampling rate must be at least double the highest frequency part of the signal. So if you have a 100 Hz signal it must be sample at 200 Hz, or one sample every 0.05s (1/200 s), or more. If not you loose the higher frequency components.

Information

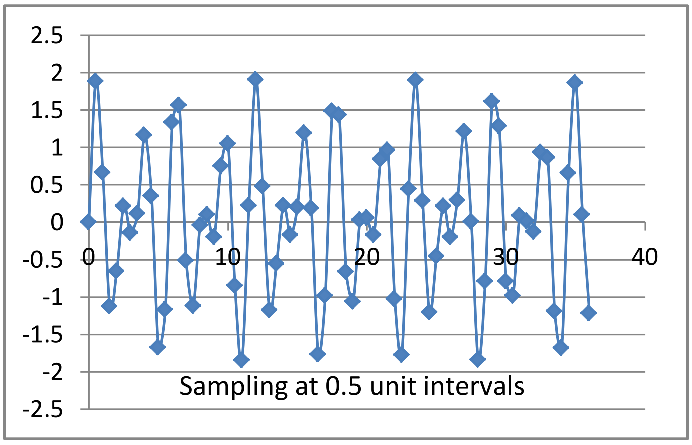
Minimum rate of sampling > 2 x maximum frequency of signal

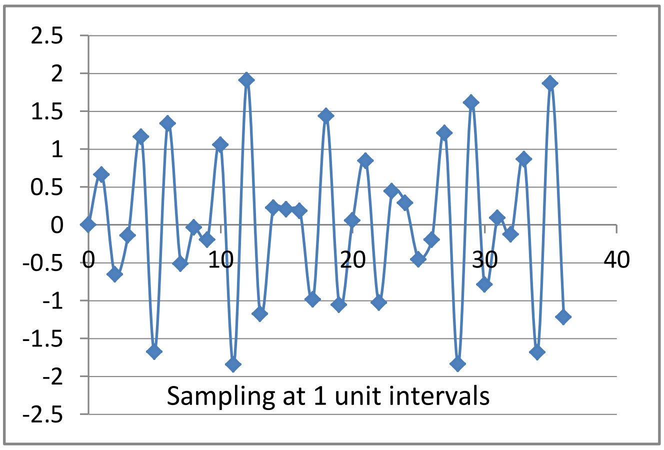
An example of a sampling rate of around four times the frequency is shown below. The sampling points are shown by the red discs and lines.

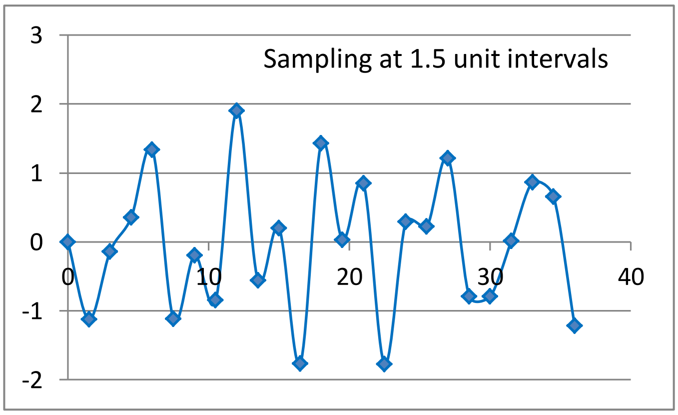


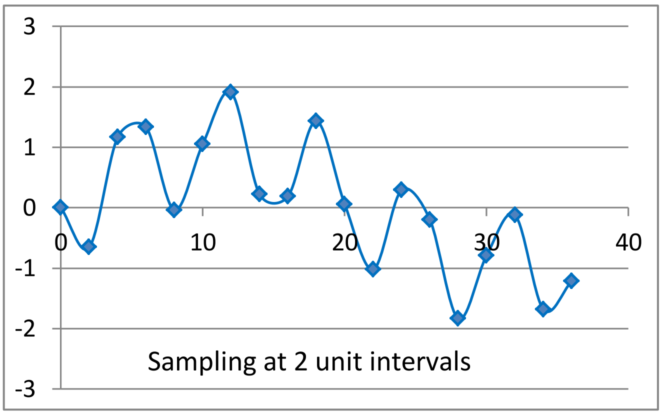
You can probably see that if the wave were reconstructed from just those few values it would not resemble the original wave at all closely! Many more sampling points would be required.

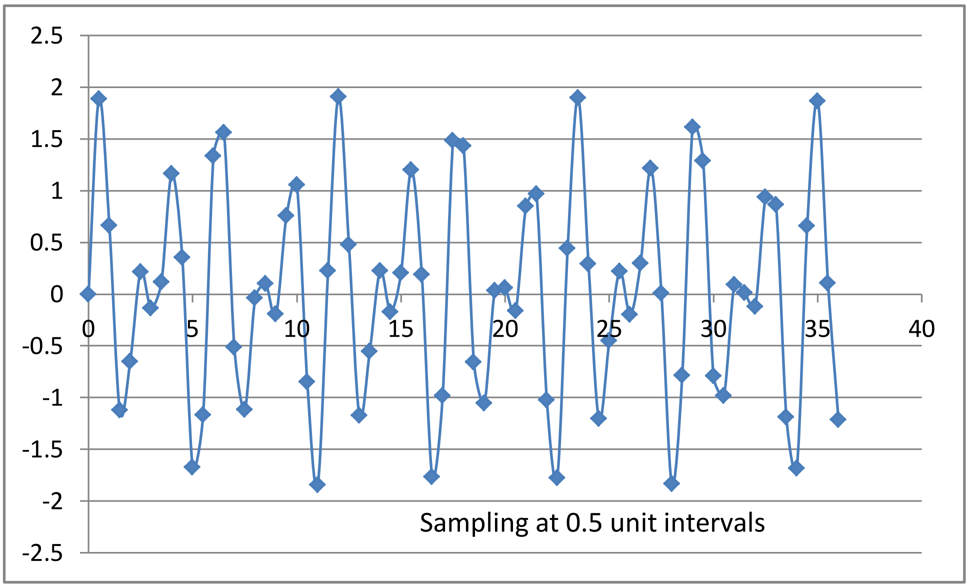
Sampling rates – examples

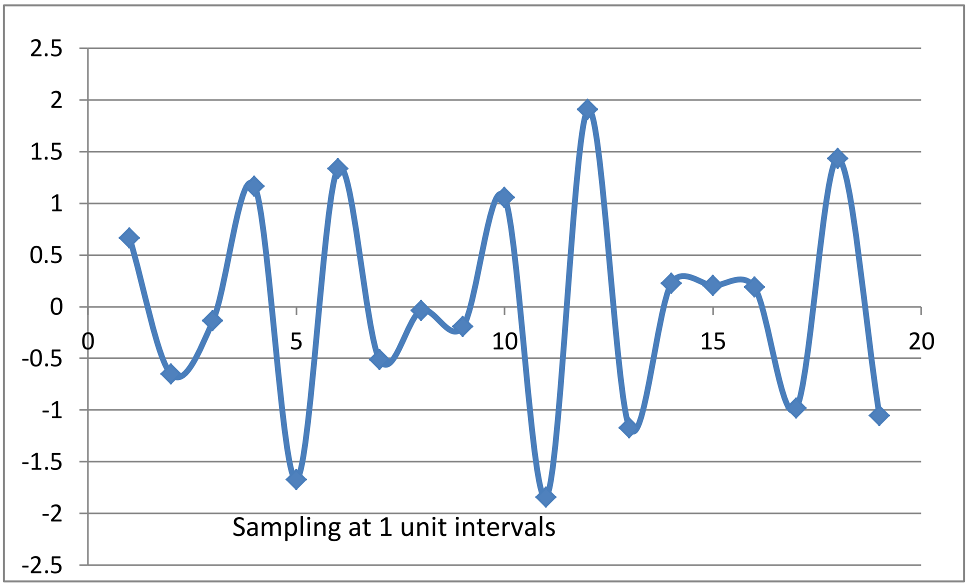


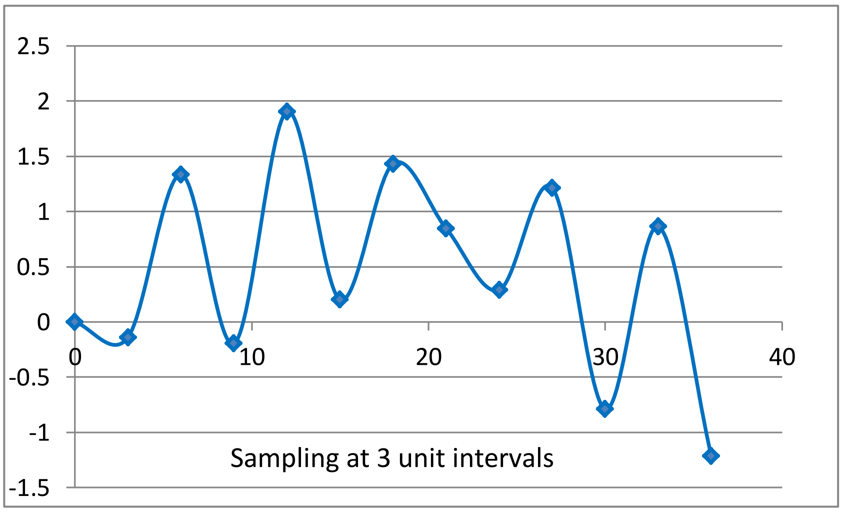










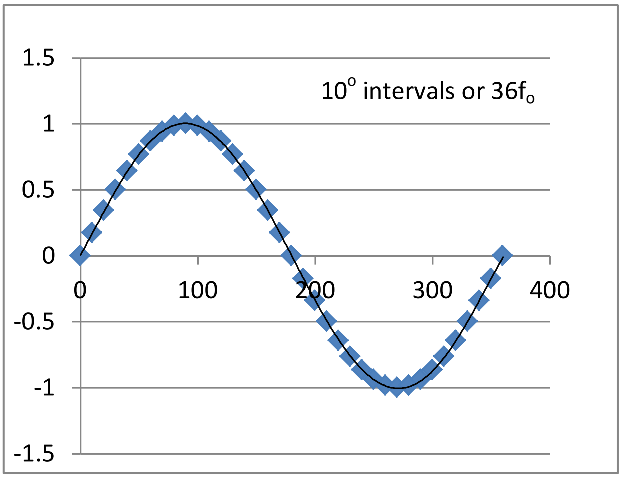


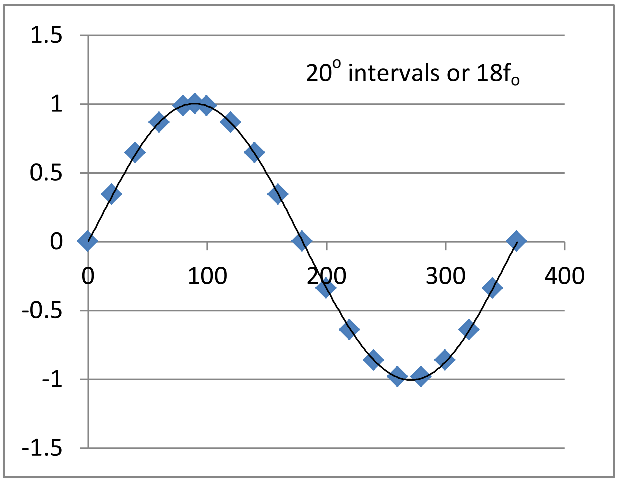
Explanation

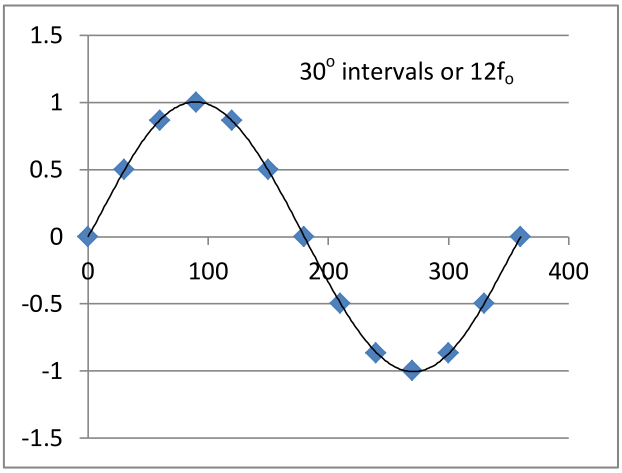
One very important thing about the reconstruction of the waveform after sampling is that it is done using a **smooth** curve.

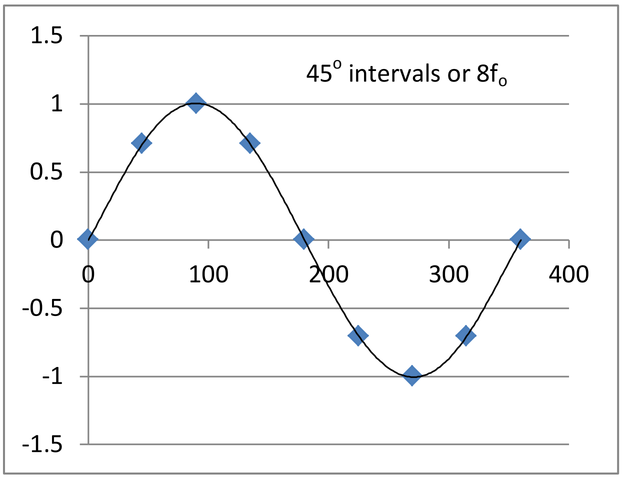
Further examples of digital sampling

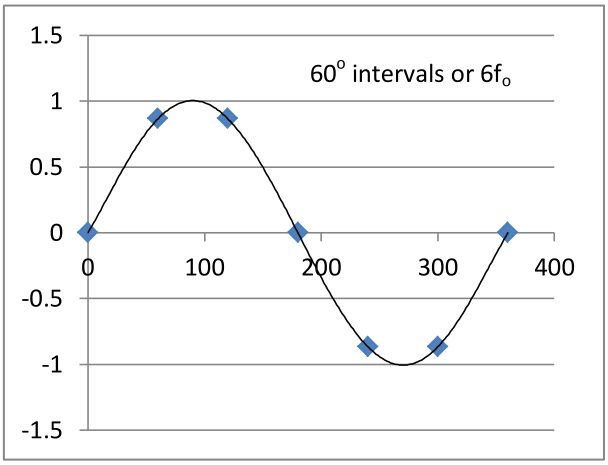
The graphs on this sheet show the waveforms produced for different sampling rates compared with the original frequency.

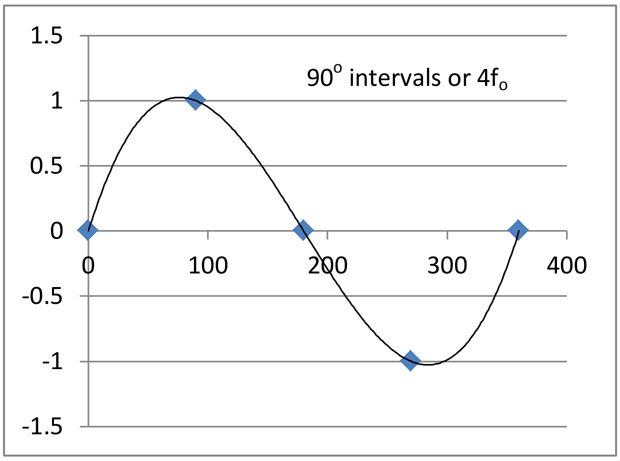


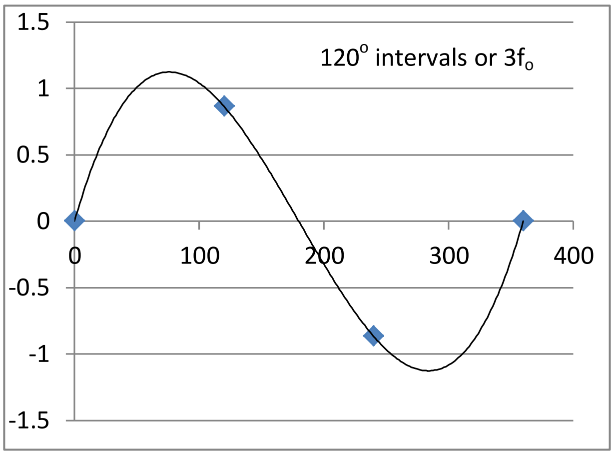


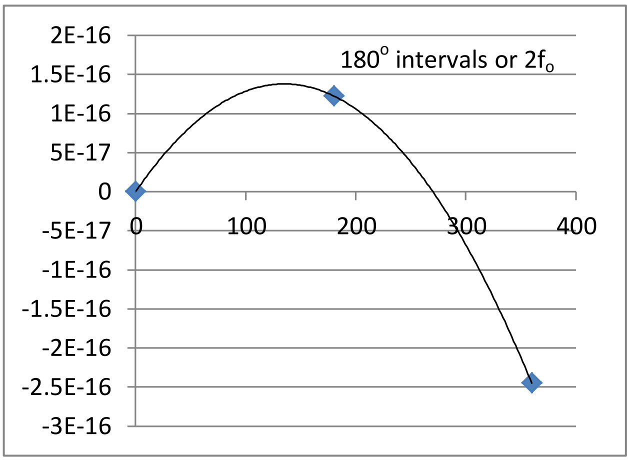


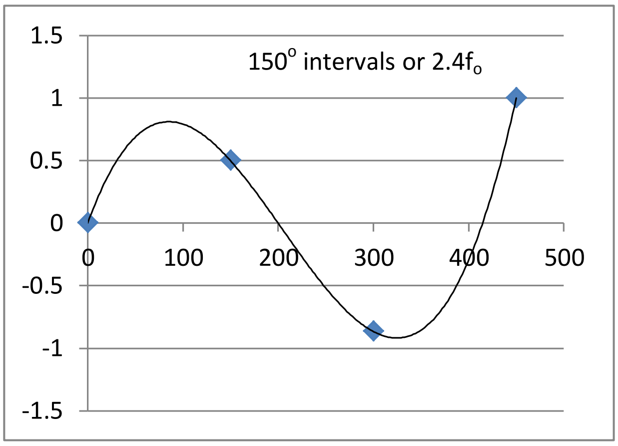


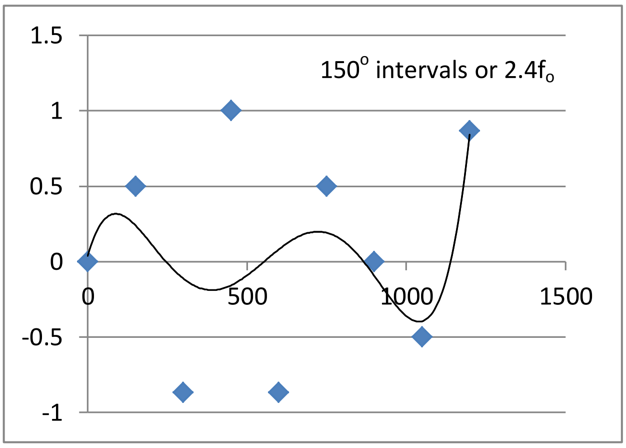








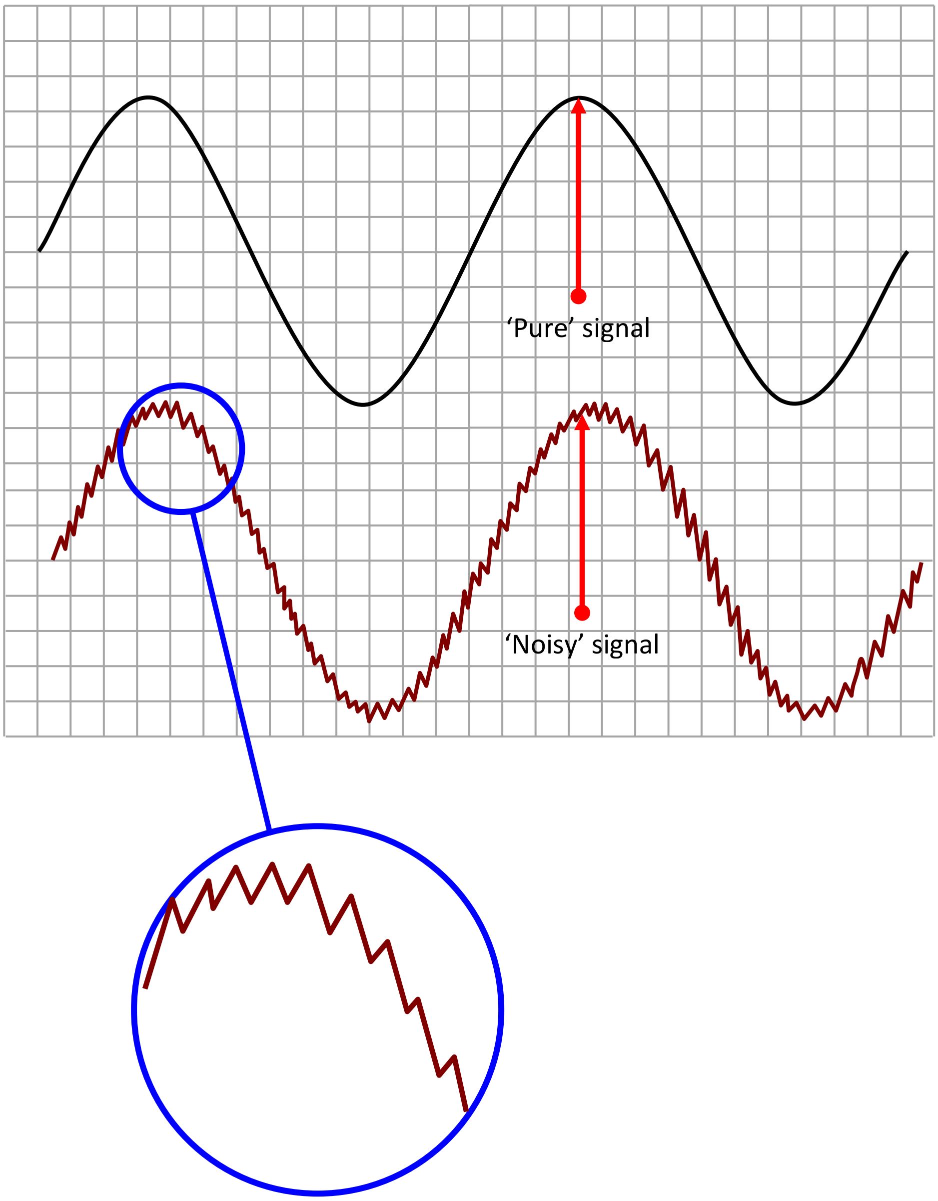




The reproduced signal tends to 'break up' below $2f_o$.

Example of a 'noisy' signal

The peak-to-peak value of the main signal is about 10.8 units while the peak-to-peak value of the noise is about 0.8 units although this does vary somewhat during the signal.



This means that the signal to noise ratio is 10.8/0.8 = 13.5

Maximum number of bits per sample that can be coded for this signal

$$= \log_2 [\text{Total voltage variation } (V_{total})/\text{Noise voltage variation } (V_{noise})]$$
$$= \log_2[13.5] = 2.6$$

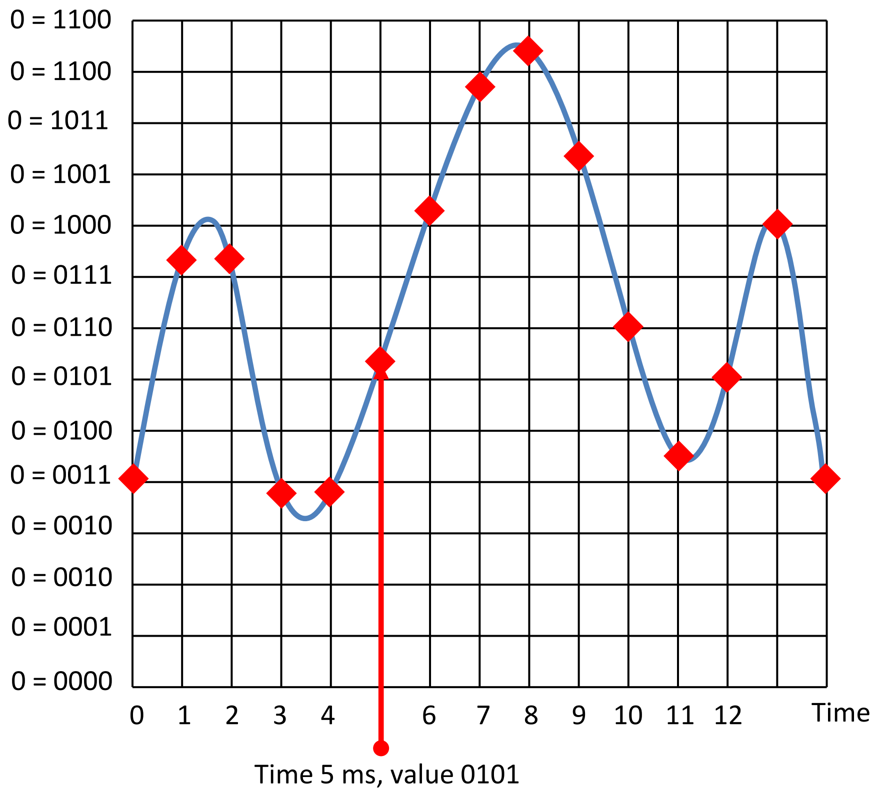
Therefore maximum number of bits per sample = 2.

Maximum number of bits per sample and the signal to noise ratio

The maximum number of useful bits per sample (b) is limited by the 'signal to noise ratio'
This is the ratio of the total voltage variation to the noise voltage variation.

Information

Maximum bits per sample (b)
= $\log_2$ [Total voltage variation (V_{total})/Noise voltage variation (V_{noise})]



The signal above has been digitised using 4 bit samples – the number of sampling levels is therefore $2^4 = 16$. These sampled values are shown by the red diamonds. The greater the number of bits per sample the better the resolution of the signal – in other words the better the reproduction of the signal. For an 8 bits per sample digitised signal there would be 2^8 (=256) sampling levels.

Example 5

Calculate the maximum bits per sample if the total voltage variation is 0.2 V and the noise signal has an amplitude of 2.5 μV

Maximum bits per sample (b) = $\log_2$ [0.2/2.5x10^{-6}] = $\log_2$ [80000] = 16.28 ≈ 16

Examples 6

(a) A noisy signal is digitised using 8 bits per sample. How many sampling levels are there?
Sampling levels = 2^8 = 256
This would 'include' a variation due to noise and so rather fewer bits per sample would have been better.

(b) If the signal to noise ratio of a waveform is 128 how many bits per sample is sufficient to code all the information in that signal?
V_{noise}/V_{total} = 128 2^7 = 128 and so 7 bits per sample is sufficient

(c) Calculate the rate of transfer of digital information if the sampling frequency is 5000 Hz.
Rate of transfer = 5000x7 = 35000 bits per second

If a signal is sampled using N bit sampling then the resolution is the peak to peak voltage divided by the number of sampling levels -1.

Information

Signal resolution = Peak to peak voltage/(Number of sampling levels -1)
= Peak to peak voltage/2^N

Resolution and sampling levels

Example 7

A signal with a peak-to-peak voltage of 200 mV is digitised using 16 bit sampling.

Number of sampling levels = 2^{16} -1 = 65536 – 1 = 65535

Resolution = 200/65535 = 0.00305 mV = 3.05 μV

Sound intensity and decibel levels

The level of sound intensity is defined by the decibel. The ratio of the intensities (I/I_o) of two sounds is given in decibels (dB) by the formula:

Information

$$\text{Decibel level} = 10 \log (I/I_o)$$

Therefore increase in intensity $(I/I_o) = 10^{(\text{decibel level increase}/10)}$

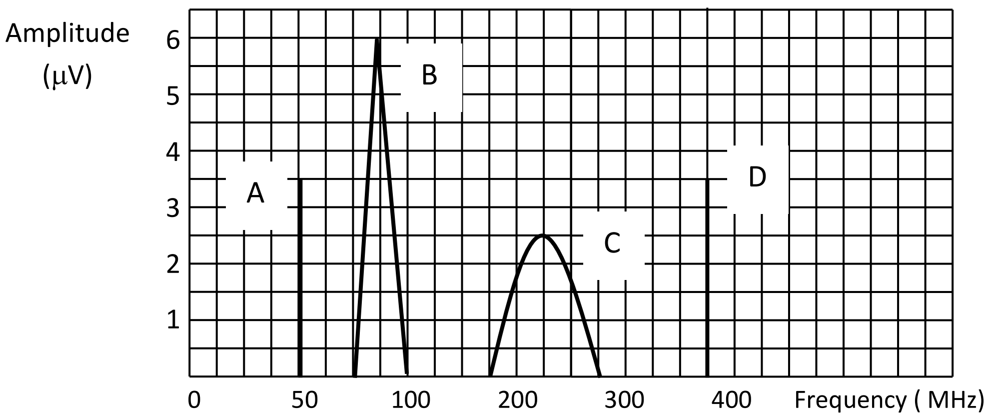
Example 8

If the intensity of the sound increases from 0 dB (I_o) to 50 dB (I) the increase in intensity is:

Increase in intensity = 10^5

Signal frequency and bandwidth

Bandwidth is the maximum spread of frequencies within the signal. The following graph shows four signals.



The signal with the greatest:
(a) frequency is D
(b) wavelength is A
(c) amplitude is B
(d) bandwidth is C

Information

Bandwidth = bit rate/2   For practical purposes: Bandwidth = bit rate

The following example is about a colour laptop screen, the number of intensity values for each subpixel within that screen and the bandwidth needed for the transmission of an image.

Example 9

An image from a telescope is displayed on a colour computer screen of 1200x1080 pixels. Each colour pixel in the screen contains a red, green and blue sub-pixel and is coded by a 12 bit number. Fifty images are displayed on the screen every second.

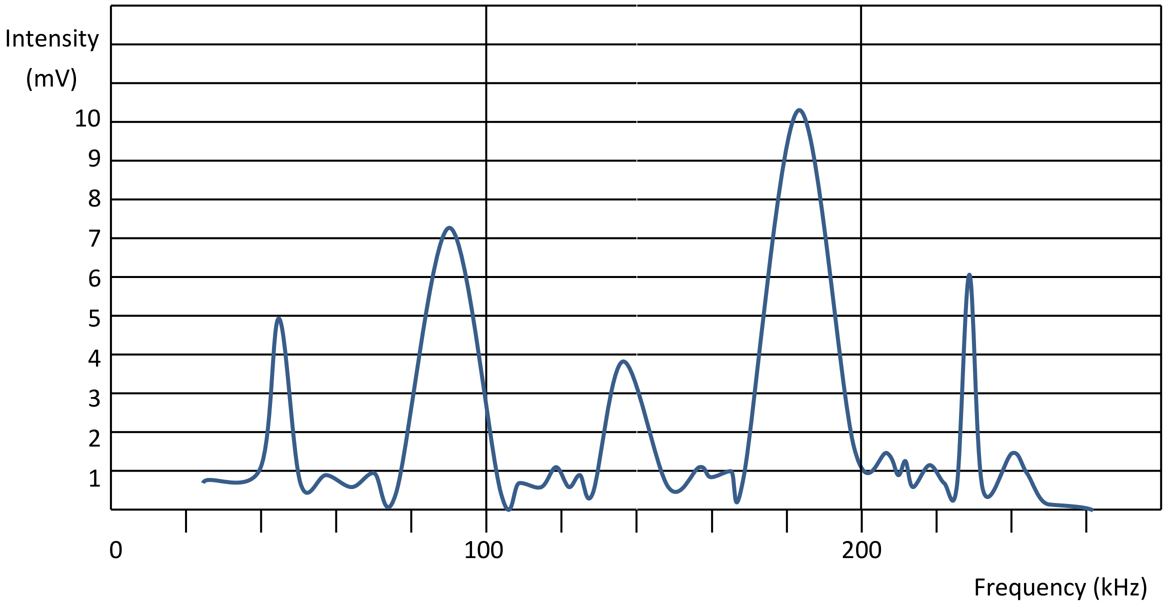
(a) the number of alternative intensity values for each sub pixel is 2^{12} = 4096

(b) the number of bits of uncompressed information are 1920x1080x12x3 = 74.6 Mbits

(c) bandwidth needed for this rate of transmission of information = bit rate/2
But practically bandwidth= bit rate = 1920x1080x3x50 = 3.73 GHz

Example 10

This example concerns the graph shown below. It shows a basic signal (frequency f_o) and four harmonics (f1, f2, f3 and f4).
Use the graph to estimate the frequency f_o as accurately as you can. Explain why your method is likely to be accurate.
Using: f_o = 45, using f_1: f_o = 90/2 = 45, using f2: f_o = 140/3 =46, using f3: f_o = 185/4 = 46, using f4: f_o = 230/5 =44.
Average value for f_o = 45 + 45 + 46 + 46 + 44 = 226/5 = 45.2 kHz ≈ 45 kHz



Quick response (QR) code table

Bar codes are one dimensional and are scanned by a marrow beam of light. The amount of digital information stored in one is limited by the length of the bar and by the number of lines within it.

QR codes are two dimensional and scanned using an image recognition device such as a digital camera with a suitable ap

.

The fact that they are two dimensional means that they can store a lot more information.

The image shown here is a 29x29 code for www.schoolphysics.co.uk.

The blocks of code at the top right and top and bottom left are used to align the image.